Measurement of the Filtration Diffusivity $D(\phi)$ of a Flocculated Suspension

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In modeling the pressure filtration of flocculated suspensions using compressional rheology, the filtration diffusivity function $D(\phi)$ plays the role of a diffusion coefficient in determining the time scale of the filtration process. Its dependence on volume fraction is an important factor in filtration process design. The volume of filtrate expressed per unit membrane is $V = \beta t^{1/2}$. The value of β depends on $D(\phi)$ and various relationships between the two that involve data differentiation have been discussed in the literature. These involve an unknown but bounded parameter. Here a physical approximation is made that justifies setting this parameter to zero. Further, a new approximate expression for β^2 in terms of $D(\phi)$ that avoids data differentiation is determined. Alternatively, the hindered settling factor can be recovered from an analytic expression involving data differentiation of β data with applied pressure. The accuracy of each of these approximations is assessed.

Introduction

It is conventional in modeling filtration processes with compressible filter cakes to use the solids flux equation

$$\phi u = - D(\phi)\nabla\phi + \phi S, \tag{1}$$

where gravity is negligible (Kirby and Smiles, 1988; Sherwood and Meeten, 1997; Landman and White, 1997). Here $\phi(r,t)$ is the local volume fraction of solids, $u(r,t)$ is the solids velocity, and $S(t)$ is the suspension (solids plus fluid) flux at time t. If $w(r,t)$ is the fluid velocity, then

$$S(t) = \phi u(r,t) + [1 - \phi(r,t)]w(r,t). \tag{2}$$

The flux $S(t)$ is spatially constant during the process and is determined by outflow boundary conditions. This is not the place for a debate on the validity of Eq. 1, which is used by many authors and is simply derived from a force balance on the solid component of the suspension (Landman and White, 1994). It relies heavily on the existence of a solids stress transmitted by the particle network if flocculated and by interparticle interactions (electrostatic, steric, dispersive) if the suspension is stable. In the latter case the solids stress is just the osmotic pressure of the particles and, due to the short-range nature of the interparticle repulsions, could be neglected until the suspension has random close packing. Until this volume fraction, resistance to compression arose solely from Darcian flow of the fluid relative to particles in the immobilized bed for stable suspensions. For flocculated suspensions, the range of solids volume fraction over which forces are transmitted through the particles can be very much larger, and the particle stress can be a significant contribution to resistance over a much longer portion of the filter press than just the close-packed piece at the membrane.

The filtration diffusivity $D(\phi)$ is given by

$$D(\phi) = \frac{(1-\phi)^2 \dfrac{dP_y(\phi)}{d\phi}}{R(\phi)}, \tag{3}$$

where $R(\phi)$ is the hindered drag coefficient of the solids per unit suspension volume (Landman et al., 1995). The compressive yield stress $P_y(\phi)$ is the maximum solids stress that the suspension network in the filter cake at volume fraction ϕ can sustain before it begins to yield and dewater. The measurement of $P_y(\phi)$ has been discussed elsewhere (Buscall and White, 1987; Landman and White, 1994; Green et al., 1996;

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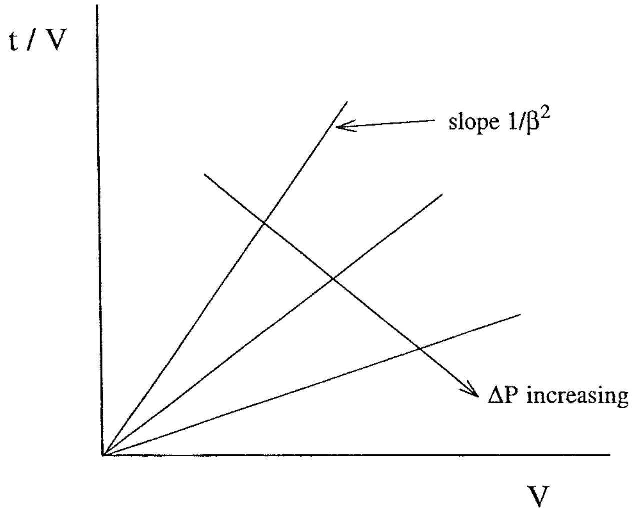

Figure 1. Straight lines *t/V* vs. *V* with slope $1/\beta^2$, where *V* is the volumetric flux at time *t* for different values of applied pressure ΔP.

Channell and Zukoski, 1997). The filtration diffusivity plays the role of a diffusion coefficient in determining the time scale of the filtration process (Landman and White, 1997). Its dependence on volume fraction is an important factor in filtration process design. The diffusivity is also important in soil science (Smiles and Harvey, 1973).

It was recognized early that the compact-bed-formation region (the short-time behavior) in pressure filtration of a suspension at constant piston pressure ΔP contained information about $D(\phi)$. When membrane resistance is negligible, the short-time solution of the filtration equations is a similarity solution and the volume of filtrate expressed per unit filter membrane area obeys

$$V = \beta t^{1/2}. \tag{4}$$

Thus a t/V vs. V plot of the early time filtration data yields a straight line of slope $1/\beta^2$ (Figure 1). It is possible to show that

$$\beta^2 = 2\int_{\phi_0}^{\phi_\infty} \frac{d\phi}{\phi^2}\left(\frac{1}{\phi_0} - \frac{1}{\phi}\right)\frac{D(\phi)}{F(\phi)} \tag{5}$$

(Smiles and Harvey, 1973), where $F(\phi)$ is the flux concentration relation. White and Perroux (1987) have discussed methods to deconvolute Eq. 5 to derive $D(\phi)$ from experimental $\beta^2(\Delta P)$ data. By assuming a form of the flux-concentration relation as

$$F(\phi) = \left[\frac{\dfrac{1}{\phi_0} - \dfrac{1}{\phi}}{\dfrac{1}{\phi_0} - \dfrac{1}{\phi_\infty}}\right]^r, \tag{6}$$

Smiles and Harvey (1973) showed that

$$\frac{\partial\beta^2}{\partial\phi_\infty} - \frac{r\beta^2}{\left(\dfrac{1}{\phi_0} - \dfrac{1}{\phi_\infty}\right)} = 2D(\phi_\infty)\left(\frac{1}{\phi_0} - \frac{1}{\phi_\infty}\right), \tag{7}$$

where ϕ_0 is the initial-volume fraction. The ϕ_∞ is the volume fraction of the filter cake at infinite time, but it is also the volume fraction at the filter membrane at all times, given by

$$\Delta P = P_y(\phi_\infty). \tag{8}$$

The parameter r arises from the assumed form for the flux-concentration relation and is not a known quantity, but is bounded $(0 \le r \le 1)$. The term in which it appears is normally quite small, and so its value is hopefully not vital to a determination of $D(\phi)$. Sherwood (1997) also derived Eq. 5, by differentiating upper and lower bounds for β^2. Sherwood's analysis does not assume a functional form for any of the system quantities, but suffers the mathematical awkwardness of being strictly not allowable, a matter discussed by Sherwood. The parameter r in this derivation is also not determinable.

In this article we make a physical approximation that is readily understood and whose validity is testable, to derive Eq. 7 with $r = 0$. This provides an approximate expression for the determination of $D(\phi)$ or alternatively $R(\phi)$ from $\beta^2(\Delta P)$ data via differentiation. In the second part of the article we use the results of Landman and White (1997) to derive a new approximate expression for β^2 as a function of $D(\phi)$, which lends itself to deconvolution in a way that does not involve data differentiation. We examine the accuracy of deconvolution of $\beta^2(\Delta P)$ data to obtain $D(\phi)$ using numerically derived data with and without Gaussian noise.

Physical Derivation of the Smiles-Harvey Formula

The geometry of the pressure-filtration experiment is shown in Figure 2. An initial volume fraction of solids ϕ_0 is filtered through a membrane under constant applied pressure ΔP. Distance from the membrane is measured by the coordinate z. The piston at time t is at $z = h(t)$. In the pressure filter there are two regions initially: a region near the piston with volume fraction ϕ_0 where solids stress is zero ($\phi_0 < \phi_g$, the gel point for the flocculated suspension), and the region near the membrane, namely $0 \le z \le l(t)$. In the former region the fluid pressure is ΔP. In the region $0 \le z \le l(t)$ the filter cake is formed and the solids stress $P_s(z,t)$ is given by

$$P_s(z,t) = P_y[\phi(z,t)]. \tag{9}$$

The filter cake varies from

$$\phi(0,t) = \phi_\infty \tag{10}$$

$$P_s(0,t) = \Delta P \tag{11}$$

to

$$\phi(l,t) = \phi_g \tag{12}$$

$$P_s(l,t) = 0. \tag{13}$$

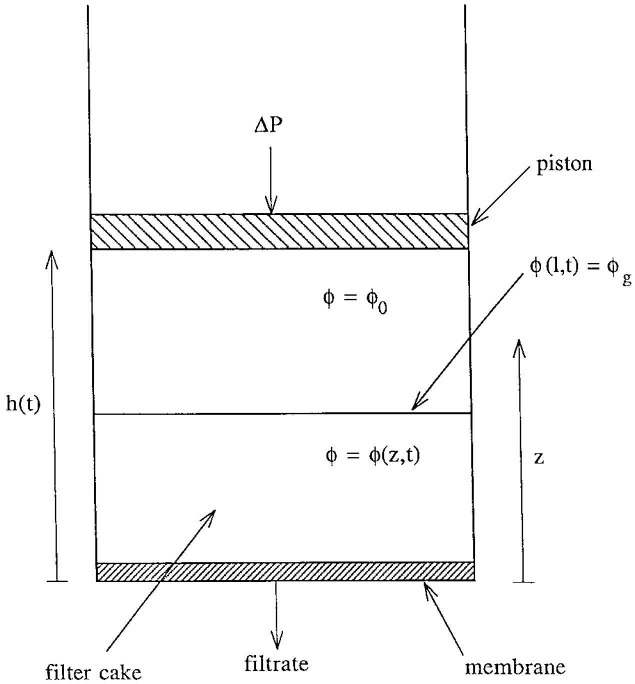

Figure 2. Geometry of a pressure filtration process.

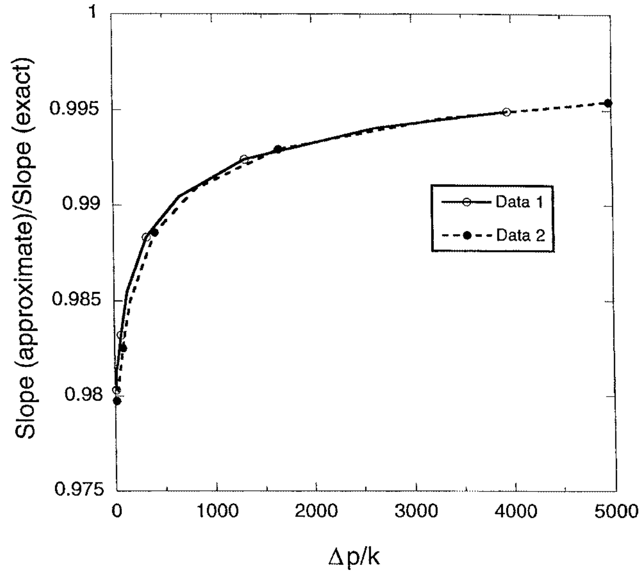

Figure 3. Ratio of the approximate slope *t/V* vs. *V* over the exact slope for two data sets in Table 1.

Values near unity indicate the validity of the approximation $u(z,t)$ is negligible compared to dh/dt in the compact bed.

In pressure filtration the suspension flux S is given by

$$S = \frac{dh}{dt}\hat{z} \tag{14}$$

(Landman et al., 1995), and Eq. 1, with $u = -u\hat{z}$, may be written in the form

$$\frac{D(\phi)}{\phi}\frac{\partial \phi}{\partial z} = u + \frac{dh}{dt}. \tag{15}$$

Landman et al. (1995) analyzed the approximation that the solids velocity u can be neglected in comparison to the piston velocity dh/dt in the compact bed $0 \le z \le l(t)$. This approximation led to predicted values of the slope β^2 that were within a percent or two of the exact values over a wide range of applied pressure ΔP using numerical model data (Figure 3). We make the same approximation here to derive the connection between β^2 and $D(\phi)$. In this approximation Eq. 15 becomes

$$\frac{\partial \Pi}{\partial z} = \frac{dh}{dt}, \tag{16}$$

where

$$\Pi(\phi) = \int_{\phi_g}^{\phi}\frac{D(\phi)}{\phi}d\phi. \tag{17}$$

Thus

$$\Pi[\phi(z,t)] = z\frac{dh}{dt} + \Pi_\infty, \tag{18}$$

where

$$\Pi_\infty = \Pi(\phi_\infty) \tag{19}$$

is the value of Π at $z = 0$ at the membrane. Note that at $z = l(t)$

$$\Pi[\phi(l,t)] = 0 \tag{20}$$

from Eq. 12 and the definition (Eq. 17). Hence from Eq. 18

$$0 = l\frac{dh}{dt} + \Pi_\infty. \tag{21}$$

Material conservation yields

$$\int_0^{l(t)}\phi(z,t)\,dz + \phi_0(h-l) = \phi_0 h_0, \tag{22}$$

where h_0 is the piston height at $t = 0$. From Eq. 18 we have

$$\phi(z,t) = \Pi^{-1}\left(z\frac{dh}{dt} + \Pi_\infty\right), \tag{23}$$

and substituting in Eq. 22, we obtain

$$\int_0^l \Pi^{-1}\left(z\frac{dh}{dt} + \Pi_\infty\right)dz - \phi_0(h_0 - h) - \phi_0 l = 0. \tag{24}$$

A change of variables in the integral yields

$$-\frac{1}{\left(\frac{dh}{dt}\right)}\int_0^{\Pi_\infty}\Pi^{-1}(u)\,du - \phi_0(h_0-h) - \phi_0 l = 0, \tag{25}$$

which is rearranged to yield

$$\frac{d}{dt}(h_0 - h)^2 = \frac{2}{\phi_0}\int_0^{\Pi_\infty}\Pi^{-1}(u)\ du - 2\Pi_\infty. \tag{26}$$

Since from Eq. 4

$$V = h_0 - h = \beta t^{1/2} \tag{27}$$

the lefthand side of Eq. 26 must equal β^2, that is,

$$\beta^2 = \frac{2}{\phi_0}\int_0^{\Pi_\infty}\Pi^{-1}(u)\ du - 2\Pi_\infty. \tag{28}$$

The integral

$$\int_0^{\Pi_\infty}\Pi^{-1}(u)\ du = \int_{\phi_g}^{\phi_\infty} d\phi\ \phi\frac{d\Pi}{d\phi} = \int_{\phi_g}^{\phi_\infty} d\phi D(\phi). \tag{29}$$

Thus

$$\beta^2 = 2\int_{\phi_g}^{\phi_\infty} d\phi\left(\frac{1}{\phi_0} - \frac{1}{\phi}\right)D(\phi). \tag{30}$$

Hence

$$\frac{d\beta^2}{d\phi_\infty} = 2\left(\frac{1}{\phi_0} - \frac{1}{\phi_\infty}\right)D(\phi_\infty), \tag{31}$$

or alternatively

$$\frac{d\beta^2}{d\Delta P} = 2\left(\frac{1}{\phi_0} - \frac{1}{\phi_\infty}\right)\frac{(1-\phi_\infty)^2}{R(\phi_\infty)} \tag{32}$$

using Eq. 3. These expressions yield what we call *analytic* expressions for either $D(\phi_\infty)$ or $R(\phi_\infty)$, as

$$D(\phi_\infty) = \frac{\dfrac{d\beta^2}{d\phi_\infty}}{2\left(\dfrac{1}{\phi_0} - \dfrac{1}{\phi_\infty}\right)} \tag{33}$$

$$R(\phi_\infty) = \frac{2\left(\dfrac{1}{\phi_0} - \dfrac{1}{\phi_\infty}\right)(1-\phi_\infty)^2}{\dfrac{d\beta^2}{d\Delta P}}. \tag{34}$$

Since Landman et al. (1995) showed that β^2 calculated assuming $u = 0$ everywhere in the filter cake is a very accurate approximation over a wide range of ΔP, it follows that Eqs. 33–34 should be very accurate. This conclusion will be tested on some simulated data. Hence the preceding argument has provided a physical basis for Eq. 7, with $r = 0$.

It should be noted that the assumption that the solids are at rest gives, in the context of Smiles and Harvey (1973), that the flux concentration relation in Eqs. 5–6 is $F(\phi) = 1$, which immediately implies that $r = 0$, and Eq. 31 follows. Our derivation of Eq. 31 does not rely on introducing a flux function formulation (Eq. 6). However, we recognize that our derivation is an alternative to Smiles and Harvey's for the case $r = 0$.

New Approximate Expression for $D(\phi)$

In Landman and White (1997) the filtration time and maximal throughput for the full nonlinear filtration model were successfully approximated by a much simpler filtration model. This involved reformulating the problem in terms of the void ratio $e = (1-\phi)/\phi$ and changing to a material coordinate scheme, so that the boundaries to the problem remained fixed for all time. The heart of the method involved replacing the reformulated nonlinear diffusion coefficient

$$\Delta(e) = \frac{\phi^2 D(\phi)}{\phi_\infty^2 D(\phi_\infty)} \tag{35}$$

(which naturally vanishes for $\phi < \phi_g$) with a simpler diffusion coefficient

$$\Delta_{\text{eff}} = \begin{cases} 0 & \phi < \phi^* \\ 1 & \phi > \phi^*, \end{cases} \tag{36}$$

where

$$\frac{1}{\phi^*} = \frac{1}{\phi_\infty} + \frac{1}{\phi_\infty^2 D(\phi_\infty)}\int_{\phi_g}^{\phi_\infty} D(\phi)\ d\phi. \tag{37}$$

(see Landman and White (1997) for full details; this corresponds to Choice 3 on page 3155). This simpler problem gives a similarity solution for the compact-bed-formation phase (as does the full nonlinear problem) followed by a separation of variables solution for the consolidation phase. In particular, the compact-bed-formation time—the time for the piston to first touch the growing filter cake, that is, $\phi(h, t) = \phi^*$—occurs at the scaled time equal to $1/(4\alpha^2)$, where

$$\frac{\dfrac{1}{\phi^*} - \dfrac{1}{\phi_\infty}}{\dfrac{1}{\phi_0} - \dfrac{1}{\phi^*}} = \sqrt{\pi}\,\alpha e^{\alpha^2}\text{erf}\ \alpha. \tag{38}$$

From Eq. 27 and Eqs. B6, B4, and B2 in Appendix B in Landman and White (1997), it follows that

$$\beta^2 = 4D(\phi_\infty)\phi_\infty^2\alpha^2 e^{2\alpha^2}\left(\frac{1}{\phi_0} - \frac{1}{\phi^*}\right)^2. \tag{39}$$

Equations 37–39 can be written to provide an iterative method to calculate $D(\phi_\infty)$ without differentiating $\beta^2(\phi_\infty)$ data, as given in Eq. 33.

If we define

$$Z(\phi_\infty) = \frac{1}{\phi_\infty^2 D(\phi_\infty)}\int_{\phi_g}^{\phi_\infty} D(\phi)\ d\phi, \tag{40}$$

then α becomes the solution of

$$\frac{Z}{\frac{1}{\phi_0}-\frac{1}{\phi_\infty}-Z}=\sqrt{\pi}\ \alpha e^{\alpha^2}\text{erf}\ \alpha \tag{41}$$

for any given Z. We then calculate $D(\phi_\infty)$ from

$$D(\phi_\infty)=\frac{\pi}{4}\beta^2(\phi_\infty)\left(\frac{\text{erf}\ \alpha}{\phi_\infty Z}\right)^2. \tag{42}$$

There are three steps to the iterative scheme provided by Eqs. 40–42:

1. Fit $\beta^2(\phi_\infty)$ data with a function of ϕ_∞ that reflects the nature of the problem (further comments later)
2. Guess an initial form of $D(\phi_\infty)$, for example, use the constant function (unity) to start, or the $D(\phi_\infty)$ calculated from Eq. 33.
3. Calculate $Z(\phi_\infty)$ from Eq. 40, solve for α from Eq. 41, and use Eq. 42 to get an updated function $D(\phi_\infty)$.

This process is continued until $D(\phi_\infty)$ converges to within a specified tolerance.

In the next section we test this method, as well as the analytic method, on some simulated data.

Test on Simulated Data

To illustrate the robustness of this iteration scheme we resort to a numerical calculation on a model system. We choose typical parameter values as given in Table 1, and choose nine values of the piston pressure $\Delta P/k$ (see Table 1 for a definition of k), so that we operate over a range of ϕ_∞ values.

The values of the (scaled) inverse slope vs. ϕ_∞ for the two data sets are determined by solving compact-bed-formation equations using a similarity variable (Landman et al., 1995). These data must be fitted with a functional form that reflects the structure of Eq. 42. First of all, we note that $Z(\phi_g)=0$, so that $(\phi_\infty-\phi_g)$ must be a factor of β^2. We tried fitting various functional forms of the type

$$\beta^2=\begin{cases} A\phi_\infty^n(1-\phi_\infty)^m(\phi_\infty-\phi_g) & \text{for Data 1}\\ A\dfrac{\phi_\infty^n(1-\phi_\infty)^m(\phi_\infty-\phi_g)}{(\phi_{cp}-\phi_\infty)^p} & \text{for Data 2}\end{cases} \tag{43}$$

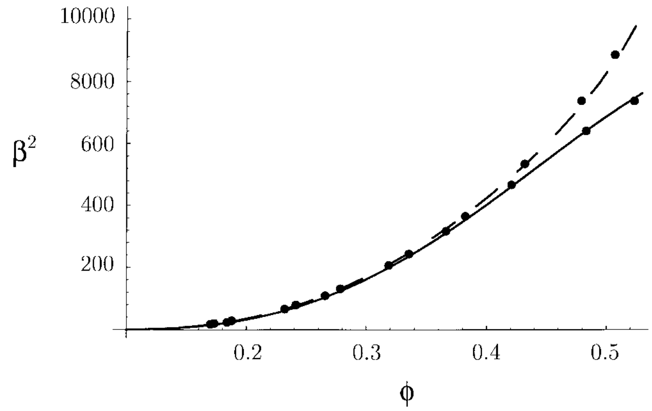

Figure 4. β^2 data vs. ϕ together with the fitted curves for Data 1 (solid line) and Data 2 (dashed line) with no noise on the data.

The general properties of the known function $P_y(\phi_\infty)$, namely, whether it is a power function or whether it approaches infinity as the volume fraction approaches a close-packing value ϕ_{cp}, has been reflected in the choice of these fitting functions. We found that for the two data sets the best fit was achieved with $n=3$ and $p=1$, and this is illustrated in Figure 4. Errors in the fits were evaluated as

$$\text{Error in } \beta^2 \text{ fit}=\sqrt{\frac{\Sigma(\beta_{\text{fit}}^2-\beta_{\text{data}}^2)}{\Sigma\beta_{\text{data}}^2}},$$

and similar errors were calculated for both the analytically and iteratively calculated $D(\phi)$ function.

The results of the simulations are summarized in Table 2 and Figures 5a–5d. With no noise, both the analytic and iterative fits approximate the curves very well for low to moderate values of ϕ and give a good approximation at the high end. We imposed 5% noise on the β^2 data and investigated the spread in the values of the fitting power m by running simulations on 100 different noisy data sets. The results in Table 2 and Figures 5c, 5d, 6a, and 6b show that both the iterative and analytic methods do achieve a fairly good approximation to the true $D(\phi)$ functions, and that the spread in the m values is reasonable when 5% noise is imposed.

We also tested the analytically calculated $R(\phi)$ from Eq. 37, and the results are summarized in Table 3 and Figures 7

Table 1. Rheological Functions for the Flocculated Suspensions

Compressive Yield Stress ($\phi>\phi_g$)	
Data 1:	$P_y(\phi)=k\left[\left(\frac{\phi}{\phi_g}\right)^5-1\right]$
Data 2:	$P_y(\phi)=k\left(\frac{(\phi/\phi_g)^4-1}{\phi_{cp}-\phi}\right)$, $\phi_{cp}=0.64$
	($\phi_g=0.1$, $\phi_0=0.05$)
Hindered Settling Factor/Permeability	
Data 1,2:	$R(\phi)=(1-\phi)^{-3.5}$

Table 2. Results of Simulated Data Test for $D(\phi)$

Noise	m	β^2 Error (%)	D_{analytic} Error (%)	$D_{\text{iterative}}$ Error (%)
Data 1				
0	2.31	1.2	4.4	8.1
5	2.33	4.9	2.4	6.6
5	2.21	4.1	9.1	12.3
5	2.28	2.0	6.4	9.9
Data 2				
0	4.56	1.9	3.3	10.3
5	4.35	1.5	13.0	2.9
5	4.48	5.8	3.6	8.0
5	4.70	3.1	7.2	15.9

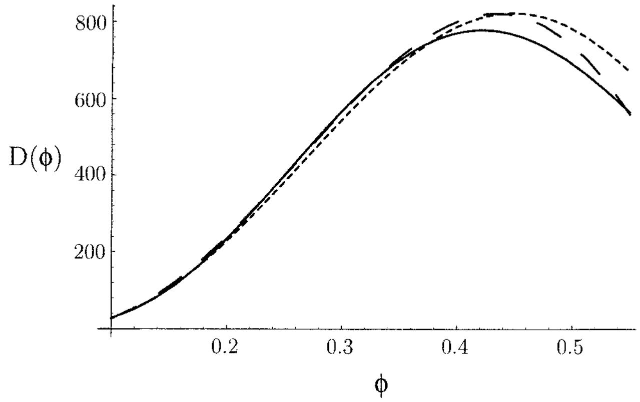

Figure 5a. $D(\phi)$ vs. ϕ for Data 1 with no noise on the data.

The exact $D(\phi)$ is the solid line, the analytically calculated $D(\phi)$ is the long ——— line, and the iteratively calculated $D(\phi)$ is the short --- line.

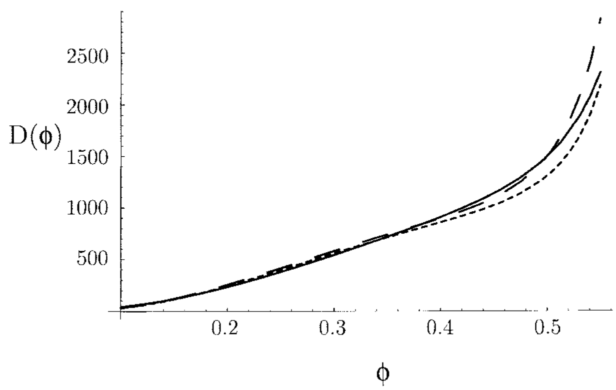

Figure 5b. $D(\phi)$ vs. ϕ for Data 2 with no noise on the data.

The exact $D(\phi)$ is the solid line, the analytically calculated $D(\phi)$ is the long ——— line, and the iteratively calculated $D(\phi)$ is the short --- line.

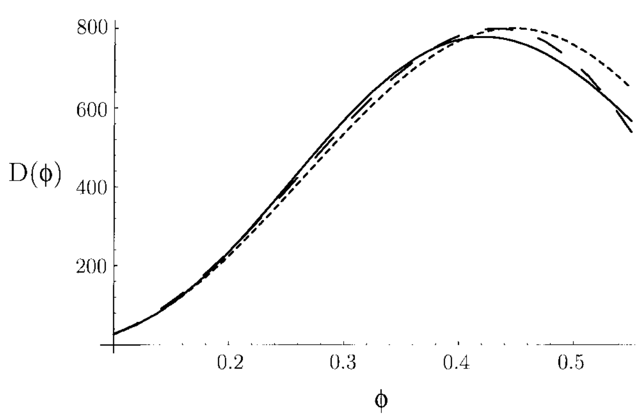

Figure 5c. $D(\phi)$ vs. ϕ for Data 1 with 5% noise on the data.

The exact $D(\phi)$ is the solid line, the analytically calculated $D(\phi)$ is the long ——— line, and the iteratively calculated $D(\phi)$ is the short ---line.

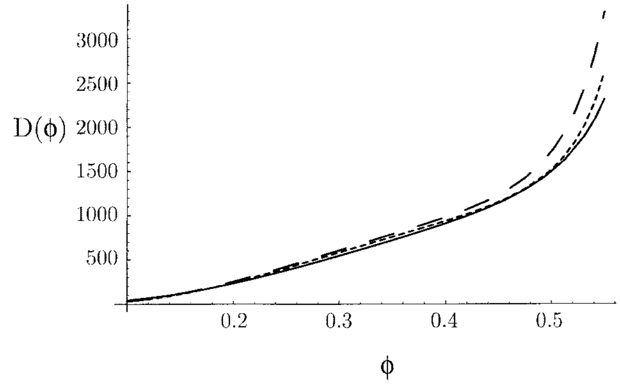

Figure 5d. $D(\phi)$ vs. ϕ for Data 2 with 5% noise on the data.

The exact $D(\phi)$ is the solid line, the analytically calculated $D(\phi)$ is the long ——— line, and the iteratively calculated $D(\phi)$ is the short --- line.

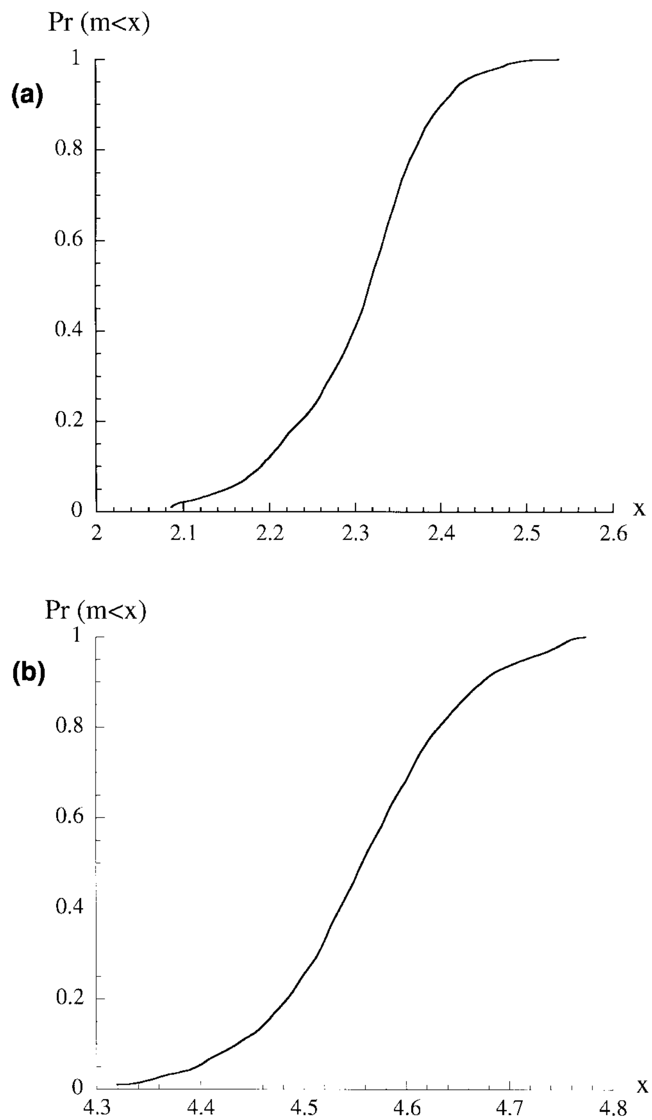

Figure 6. Cumulative probability for the value of the fitting parameter *m* for (a) Data 1 and (b) Data 2.

Table 3. Results of Simulated Data Tests for $R(\phi)$

Noise	β^2 Error (%)	$R_{analytic}$ Error (%)
Data 1		
0	0.2	3.1
5	7.3	6.7
5	5.7	14.4
5	7.8	10.7
Data 2		
0	0.3	2.6
5	2.7	8.6
5	1.7	4.2
5	4.3	6.5

and 8. First we find a fitting curve to the β^2 data as a function of $\Delta P/k$. We found an excellent fit was achieved with

$$\beta^2 = A + B\sqrt{\frac{\Delta P}{k} + E} + C\frac{\Delta P}{k}$$

for fitting parameters A, B, C, E. It should be noted that for moderately large values of $\Delta P/k$, it is adequate to set $E = 0$ in the fitting curve just given. However, for the low values of ϕ this approximation is *not* adequate, since the derivative $d\beta^2/(d(\Delta P/k)) \to \infty$ and the approximation Eq. 34 would give $R(\phi) \to 0$ as $\phi \to \phi_g$. However the definition of $R(\phi)$ being a hindered settling factor requires it to be a strictly positive monotonically increasing function for all ϕ, such that $R(\phi) \to \lambda/V_p$ as $\phi \to 0$. Here V_p is the average particle volume and λ is the Stokes drag parameter on an average particle.

For this analysis we use the four-parameter fit and we see that the iterative and analytic approximations to $R(\phi)$ for both data sets are excellent (Figure 8a). When 5% noise is applied to the data, the recovery of the $R(\phi)$ function is good (Figures 8b and 8c). In these figures, three different runs with randomly generated 5% noise are illustrated.

Conclusions

In modeling the pressure filtration of flocculated suspensions using compressional rheology, the filtration diffusivity

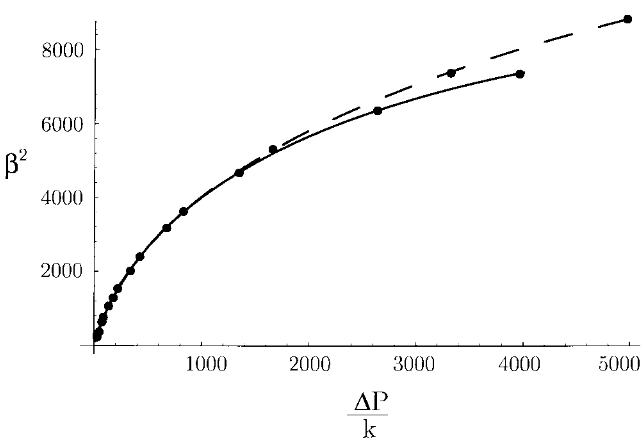

Figure 7. β^2 data vs. $\Delta P/k$ together with the fitted curves for Data 1 (solid line) and Data 2 (dashed line) with no noise on the data.

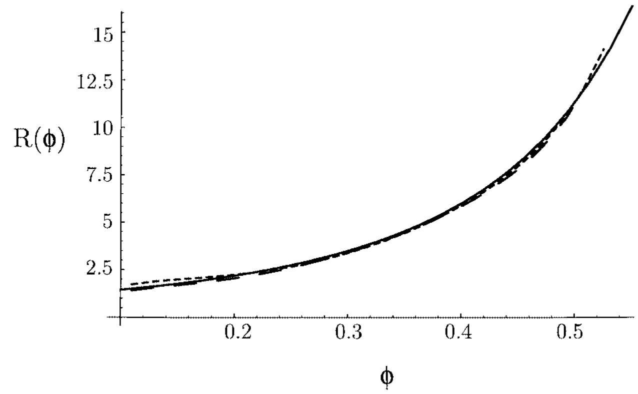

Figure 8a. $(V_p/\lambda)R(\phi)$ vs. ϕ for Data 1 and 2 with no noise on the data.

The exact function is the solid line, the analytically calculated $R(\phi)$ for Data 2 is the long ——— line, and the analytically calculated $R(\phi)$ for Data 1 is the short --- line.

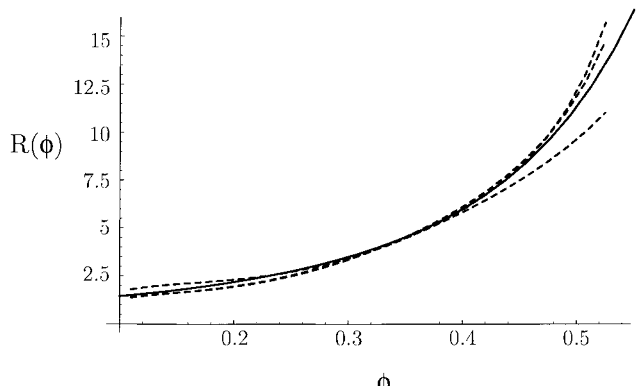

Figure 8b. $(V_p/\lambda)R(\phi)$ vs. ϕ for Data 1 with 5% noise (3 runs).

The exact function is the solid line, and the analytically calculated function is the dashed lines.

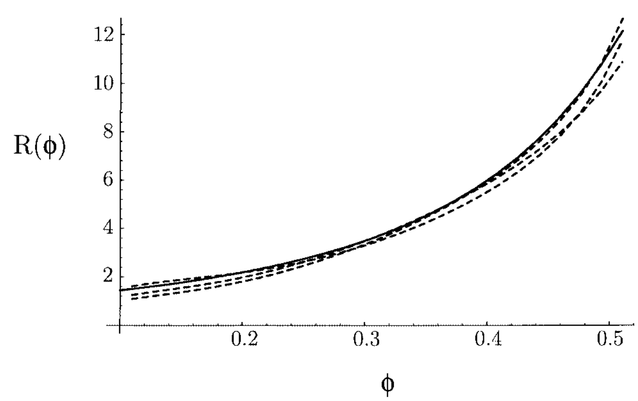

Figure 8c. $(V_p/\lambda)R(\phi)$ vs. ϕ for Data 2 with 5% noise (3 runs).

The exact function is the solid line, and the analytically calculated function is the dashed lines.

function $D(\phi)$ plays the role of a diffusion coefficient in determining the time scale of the filtration process. Its dependence on volume fraction is an important factor in filtration-process design. The volume of filtrate expressed per unit membrane is $V = \beta t^{1/2}$. The value of β depends on $D(\phi)$ and various relationships relating the two, involving data differentiation, have been proposed previously—these involve an unknown but bounded parameter. Here we make a physical approximation that the solids velocity throughout the pressure filter is much smaller than the velocity of the piston. This justifies setting this parameter to zero, hence confirming the often used analytic approximation to the diffusion function. We also obtain an analogous expression for obtaining the hindered settling function.

However, the expression for $D(\phi)$ involves differentiating noisy data—to avoid data differentiation we also give a new approximate expression for β^2 in terms of $D(\phi)$. The accuracy of these two approximations has been investigated and the methods offer promising new techniques for engineering practice.

The recovery of $R(\phi)$ is more accurate than $D(\phi)$ because the whole of the functional dependence on the compression yield stress is removed from the equation. If $P_y(\phi)$ is well characterized for the system at hand, then it is quite easy to recover $R(\phi)$. However, for understanding the dynamic behavior of a pressure filter or the optimal filtration time and throughput, the function $D(\phi)$ is required. Unless it is determined as a single quantity, it must be calculated by first differentiating $P_y(\phi)$ and combining it with $R(\phi)$.

The experimental determination of $R(\phi)$ and $D(\phi)$ for a variety of systems using these techniques is currently being undertaken by experimental groups at the University of Melbourne (Aziz et al., 1999; Usher et al., 1999) and at the University of Urbana.

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